STUDY OF HEAT TRANSFER IN A ROTARY HEAT EXCHANGER BY THE METHOD OF OPTIMIZATION OF PARAMETERS

G. M. Dolgikh, A. G. Bol'shakov, and V. F. Anikin

UDC 536.242.001.2:536.24.083

A method is presented for studying heat transfer using the optimization of parameters, realized in a study of heat exchange with a rotating cylindrical surface.

Heat exchangers with rotating heat-transfer devices are beginning to be introduced in many branches of industry, such as the power, chemical, and automotive industries [1-3]. The use of such heat exchangers for the heating and cooling of liquids seems advisable because of the possibility of considerable intensification of heat transfer from both sides of a rotating wall [2, 4]. In addition, for rotating elements it is easier to provide conditions under which contamination of the surface is inhibited or entirely absent [5-7], whereas the latter is considered to be a main problem in heat transfer [8-10]. The planning of such heat exchangers is difficult, however, because of the absence of sufficiently reliable information on heat transfer in channels which differ in numerous structural properties.

The design of an experimental rotary heat exchanger and a diagram of the experimental installation are shown in Fig. 1. The heat exchanger is intended predominantly for the cooling of viscous liquids and saturated solutions and consists of a hollow cylinder rotating coaxially in a frame with longitudinal perforated ribbing. One or two built-in pumps are mounted at the ends of the cylinder for the pumping of liquid through the annular channel and the prevention of leakage through the upper sealing unit. Since the rotation stabilizes the flow within the cylinder, a perforated wall for the distribution of the heat-transfer agent over the inner surface of the cylinder and a false bottom for bringing it out near the cylindrical surface are mounted in the cylinder in order to intensify the heat transfer. Film flow of the heat-transfer agent exists due to the false bottom in the cylinder, with the film being continuously turbulized by jets which uniformly bathe the heat-transfer surface.

The heat exchanger investigated had the following dimensions: $r' = 0.06475$ m, $b = 0.02425$ m, $l = 0.498$ m; gap between edges of ribbing and cylinder 0.1b; ribbing perforation 38.5%; gap between rim of false bottom and inner surface of cylinder 0.002 m; thickness of heat-transfer wall 0.00295 m. The performated wall was made with 234 openings arranged uniformly in staggered order in 39 rows, and the distance from the points of discharge of the jets to the surface was 0.0238 m. The heat-transfer agent was distributed within the cylinder as follows: 10% of the entire flow entered in the top section, creating a stable film, while the remaining flow was distributed over 222 openings, the diameters of which were calculated from the condition of uniform distribution.

Heat exchange in a smooth annular channel containing an inner rotating cylinder has been studied by many authors, a basic list of whom is presented in the monographs [1, 2, 11, 12]. Heat transfer through the annular gap from the wall of the rotating cylinder (or the other way) in the absence of an axial stream is studied in the majority of the works. Few experimental studies have been conducted with a determination of the heat-transfer coefficients and in the presence of an axial stream [3, 4, 13, 14]. The procedure of intensification of the heat exchange by installing longitudinal (unperforated) ribbing on the wall of the stationary cylinder was studied in [4]. Generalizing data are not obtained by the author, however. Data on the intensification of heat transfer using the jet-film motion described above are absent from the literature.

Odessa Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 1, pp. 31-39, January, 1976. Original article submitted July 19, 1974.

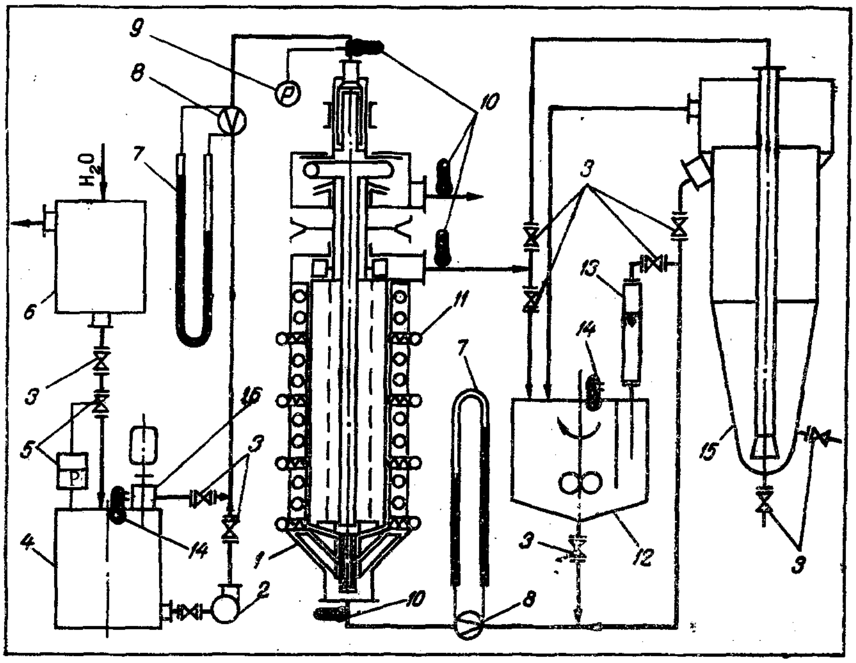

Fig. 1. Diagram of laboratory installation for the study of heat transfer in a rotary heat exchanger: 1) heat exchanger; 2) pump; 3) valve; 4) coolant thermostat; 5) floating level regulator; 6) intermediate vessel; 7) differential manometer; 8) diaphragm; 9) manometer; 10) mercury thermometer; 11) resistance thermometer; 12) thermostated vessel for preparation and regeneration of hot heat-transfer agent (initial solution for crystallization); 13) rotameter; 14) contact thermometer; 15) crystallizer containing fluidized bed; 16) thermostat pump.

It should be noted that the published reports contain considerable disagreements [3, 14, 15], evidently connected with the purely technological difficulty of the investigation of heat exchange with a rotating surface. The majority of the authors have studied heat transfer through the measurement of the temperature of the rotating wall using thermocouples or resistance thermometers and various types of current-recording devices, on the quality of which rather strict accuracy requirements are imposed [16]. Some authors [13] have measured the wall temperature after stopping the rotation, others have used condensing water vapor on the other side of the wall, while in [17] it was not possible to separate the heat-transfer coefficients at all.

In connection with the difficulty of measuring the temperature of a rotating wall, for the study of the average heat-transfer coefficients we used the method of separation of the heat-transfer coefficients with direct obtainment of the empirical dependence for one of them and of numerical values for the other through optimization of the parameters of the unknown dependence. The method is based on the assumption that the form of the empirical dependence is assumed to be known for at least one of the heat-transfer coefficients, and it is analogous in nature to Wilson's graphic method [18], but due to the computer method of analysis of the experimental data it allows one to solve accurately enough problems of higher dimensionality.

The essence of the method consists in the solution of a system of equations, constructed from experimental results, each of which consists of a sum of thermal resistances expressed in the form of assumed empirical functions:

$$c_i = \frac{1}{K_i} - \frac{\delta}{\lambda_\delta} = \frac{1}{\alpha_i'} + \frac{1}{\alpha_i''} = \frac{1}{f(\mathrm{Nu}')_i} + \frac{1}{f(\mathrm{Nu}'')_i}.$$

If from a theoretical analysis and preceding studies the form of the empirical functions is sufficiently well known for the channels on both sides of the heat-transfer wall, then the experiments are conducted so as to realize all the unrepeated combinations of the independent variables with the selected spacing in the selected range of studies. In this case the constants of the unknown equations are the result of the solution. If nothing is known about the heat-transfer coefficients on either side of the wall, then the experiments are performed in series with constant values of the coefficients in each series, which are also determined through calculation.

Since the relative errors in the determination of α' and α'' are expressed by the equation

$$\varepsilon(1/\alpha_i') = \frac{\sqrt{(\Delta 1/K_i)^2 + (\Delta 1/\alpha_i'')^2}}{1/K_i - 1/\alpha_i''},$$

$$\varepsilon(1/\alpha_i'') = \frac{\sqrt{(\Delta 1/K_i)^2 + (\Delta 1/\alpha_i')^2}}{1/K_i - 1/\alpha_i'},$$

to achieve the minimum error in the determination of both functions (or of the function for one of the heat-transfer coefficients and the numerical values for the other) one must have equality of the ranges of variation of α' and α''. If the determination of the function for Nu' is the task of the investigation, then one must have $\alpha' < \alpha''$, and vice versa.

With turbulent flow containing macrovortices without an axial stream and with developed turbulent flow containing macrovortices in the presence of an axial stream the following equation is considered the most acceptable for a smooth annular channel [2, 15]:

$$\mathrm{Nu}^* = 0.092\,(\mathrm{Ta}^2\,\mathrm{Pr})^{1/3}. \qquad (1)$$

The characters of the streams in channels with ribbing and without it are not similar to one another [4]. Whereas without ribbing the turbulent stream becomes spiral and longitudinal waves appear owing to the rotation, in the presence of ribs high turbulence is established at first at their inner edges and then spreads rapidly to the entire annular gap with an increase in the rotation rate, while longitudinal waves are not observed.

An analysis of the studies of [15, 19], however, allows one to assume that in the presence of ribbing the heat-transfer coefficient α' should not depend on the width of the gap in the region of developed turbulent flow. Since nothing is known about the form of the dependence for α'' and at the same time α' and α'' must depend on the rotation rate, the experiment can be carried out in series of tests differing from one another in the frequency of rotation of the cylinder. In each series α'' = const, for which the flow rate and temperature of the heat-transfer agent as well as the heat flux are kept constant. Moreover, with the developed turbulent-vortex mode α' does not depend on the velocity of the axial stream. Consequently, for the variation of α' in a wide enough range in each series for the outer annular channel it is necessary to use a heat-transfer agent with variable thermophysical properties. The heat exchanger studied was intended in the future for the cooling of saturated solutions in a circulating circuit containing a crystallizer of the Kristall type (Fig. 1) and as a built-in unit in a crystallizer with a circulation pipe and a baffle plate. Therefore, as the heat-transfer agents for the outer annular channel we chose aqueous solutions of glycerin with concentrations of from 0 to 70%, which have about the same range of variation of thermophysical properties as the various types of crystallizing solutions with the limits of variation of the Taylor and Prandtl numbers being $7.88 \cdot 10^3$–$1.33 \cdot 10^5$ and 4.4–65.8, respectively. In all the tests the heat flux was directed into the cylinder and comprised 64,000 W/m^2. Water with a temperature of 10.2°C and a flow rate of 1.8 m^3/h served as the coolant. We performed 80 tests in five series with a cylinder rotation frequency of 7.0, 9.0, 11.0, 13.0, and 15.0 sec^{-1}.

The variation in the stream temperature along the length of the annular channel, measured with miniature resistance thermometers, was slight (2-6°C) and practically linear. Therefore, the temperature head was determined as an arithmetic mean and the thermophysical properties entering into the Nusselt, Taylor, and Prandtl numbers were taken at the average stream temperature.

If in the proposed equation

$$\mathrm{Nu}' = A\,\mathrm{Ta}^{x_1}\,\mathrm{Pr}^{x_2}\,(\mathrm{Pr}/\mathrm{Pr}_{wa})^{x_3}$$

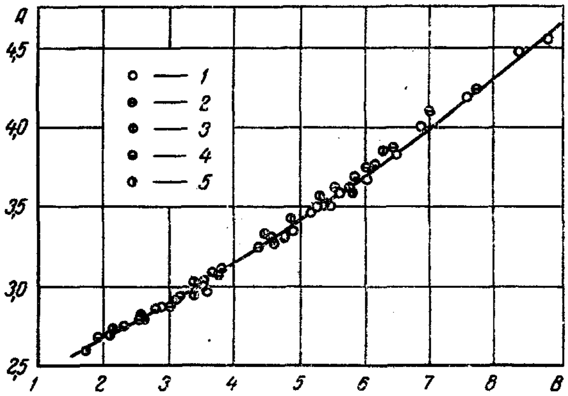

Fig. 2. Dependence of the sum of thermal resistances of heat transfer $A = (1/\alpha' + 1/\alpha'')$, $(m^2\cdot°K/W)\cdot 10^4$, in a rotary heat exchanger on the complex $B = 2b/(Ta^{2/3}Pr^{1/3}\lambda)$, $(m^2\cdot°K/W)\cdot 10^5$. ω, rad/sec: 1) 44.0; 2) 56.5; 3) 69.2; 4) 81.5; 5) 94.4; α'' = const when ω = const; G'' = 0.5 kg/sec; t'' = 10.2°C; b/r = 0.375; r = 0.06475 m; q = 64,000 W/m^2.

for the outer annular channel one takes $x_1 = {}^2/_3$ and $x_2 = {}^1/_3$ in accordance with Eq. (1), then for each series of functions with $(Pr/Pr_{wa}) = 1$

$$c=\left(\frac{1}{\alpha'}+\frac{1}{\alpha''}\right)=\frac{1}{A}\cdot\frac{2b}{Ta^{2/3}Pr^{1/3}\lambda}+\frac{1}{\alpha''} \tag{2}$$

should represent the equation of a straight line. The experimental points in Fig. 2 show that the function (2) is continuous in the investigated range. It deviates from a straight line, however, with the deviation increasing with a decrease in α'. Since the variation in Pr was considerable, it is natural to assume that the observed deviation appears because of the neglected $(Pr/Pr_{wa})^{x_3}$ and to take $x_1 = {}^2/_3$ and $x_2 = {}^1/_3$ as correct.

Using α'' one can express the properties of water and glycerin solutions at the wall temperature by the equations

$$\mu'_{wa}=\mu'\exp\left[B\left(\frac{1}{T''+q\,\frac{\delta}{\lambda_\delta}+\frac{q}{\alpha''}}-\frac{1}{T'}\right)\right]; \tag{3}$$

$$\lambda'_{wa}=\lambda'-\beta_\lambda\left(T-T''-q\,\frac{\delta}{\lambda_\delta}-\frac{q}{\alpha''}\right); \tag{4}$$

$$C'p_{wa}=C'p-\beta_c\left(T'-T''-q\,\frac{\delta}{\lambda_\delta}-\frac{q}{\alpha''}\right). \tag{5}$$

Furthermore, it is seen from Fig. 2 that with an error of ±4% the function (2) does not depend on the rotation rate, although this does not follow from the formulation of the experiment. Therefore, by introducing $(Pr/Pr_{wa})^{x_3}$ into Eq. (2) and using Eqs. (3)-(5) with $x_1 = {}^2/_3$, $x_2 = {}^1/_3$, and $x_3 = {}^1/_4$ we obtain a system of 80 equations containing six unknowns (A, α_1'', α_2'', α_3'', α_4'', α_5''), consisting of five groups containing two unknowns each (A and α_1'', A and α_2'', A and α_3'', etc.), and all the equations of the system have the form

$$c^i_j-\frac{1}{(\alpha'')^i_j}=\frac{\lambda^i_j}{2bA\,(Ta^i_j)^{2/3}(Pr^i_j)^{1/3}}\left\{\frac{Cp^i_j-\beta^i_{cj}\left[(T')^i_j-(T'')^i_j-q^i_j\,\frac{\delta}{\lambda_\delta}-\frac{q^i_j}{(\alpha'')^i_j}\right]}{\lambda^i_j-\beta^i_{\lambda j}\left[(T')^i_j-(T'')^i_j-q^i_j\,\frac{\delta}{\lambda_\delta}-\frac{q^i_j}{(\alpha'')^i_j}\right]}\times\right.$$
$$\left.\times\frac{\mu^i_j}{Pr^i_j}\exp\left[B^i_j\left(\frac{1}{(T'')^i_j+q^i_j\,\frac{\delta}{\lambda_\delta}+\frac{q^i_j}{(\alpha'')^i_j}}-\frac{1}{(T')^i_j}\right)\right]\right\}^{1/4} \tag{6}$$

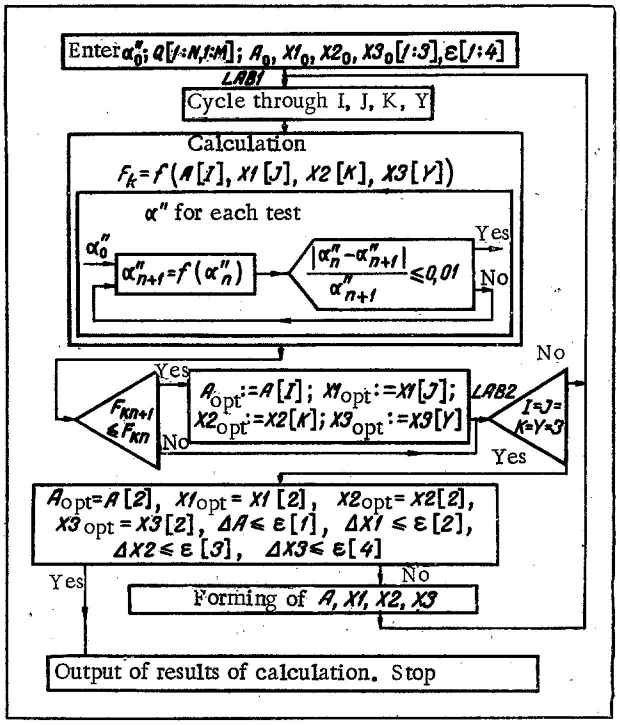

Fig. 3. Block diagram of program for solution of the problem.

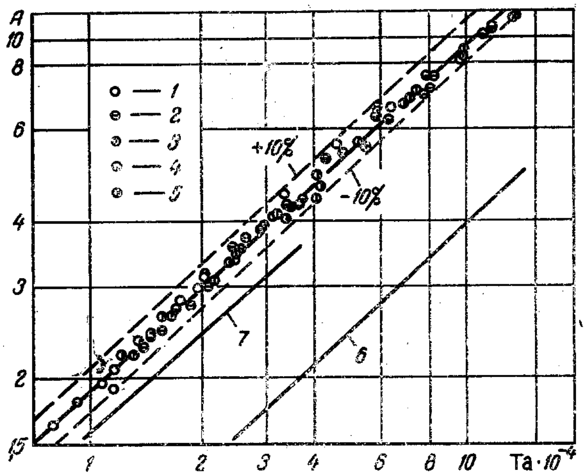

Fig. 4. Comparison of experimental values of Nusselt number with values calculated by Eq. (8) ($A = 10^{-2} \cdot Nu'/[Pr^{1/3}(Pr/Pr_{wa})^{1/4}]$); 1-5) the same as in Fig. 2; 6) by Eq. (1); 7) from data of [4].

TABLE 1. Experimental Results by Series of Tests

Indices	ω, rad/sec				
	44,0	56,5	69,2	81,5	94,4
n	15	18	17	17	13
α'', $W/m^2 \cdot °K$	4692	4675	4657	4638	4721
$\bar{\varepsilon}_{Nu'}$, %	4,4	4,2	3,3	3,3	4,8
$(\varepsilon_{Nu'})_{max}$, %	10,4	6,5	7,0	5,4	8,9
$\bar{\varepsilon}_{\alpha''}$, %	3,1	2,1	1,4	1,2	1,5
$(\varepsilon_{\alpha''})_{max}$, %	9,4	5,2	2,0	2,5	2,5

If the system of Eqs. (6) is solved by minimizing the values of the function

$$F_k = \sum_{j=1}^{m} \sum_{i=1}^{n} \left[\frac{(\alpha'')_j^i - \bar{\alpha}_j''}{\bar{\alpha}_j''} \right]^2, \tag{7}$$

then the problem of organization is simplified to one unknown quantity A. Then α'' in each equation is determined by an iteration process with selected values of A.

A block diagram of the program for the solution of the problem through optimization by scanning over A, x_1, x_2, and x_3 with a quality function determined by Eq. (7) is shown in Fig. 3, while the results of the solution for the series of tests with the assigned values $x_1 = {}^2/_3$, $x_2 = {}^1/_3$, and $x_3 = {}^1/_4$ are shown in Table 1. The coefficient A equals 0.41 with a root-mean-square relative error of 4% with respect to α' and a root-mean-square relative error of 2% with respect to α''. The equation

$$Nu' = 0.41\, Ta^{2/3}\, Pr^{1/3} (Pr/Pr_{wa})^{1/4} \tag{8}$$

obtained satisfies with a maximum error of 10% all the experimental points (Fig. 4), determined from the equations

$$\frac{1}{(\alpha'_{ex})_j^i} = \frac{1}{c_j^i} - \frac{1}{\bar{\alpha}_j''}. \tag{9}$$

For a comparative evaluation of Eq. (8) in Fig. 4 we show Eq. (1) in the coordinates $Nu'/(Pr)^{1/3}$—Ta where Nu' is calculated as Nu' = 2Nu* using [2, 13].

It is interesting to note that the use of perforated ribbing intensifies the heat transfer in the annular channel by two times in comparison with channels without ribbing [2, 15] and by 25-30% in comparison with unperforated ribbing [4].

The considerable intensification of heat transfer with the help of perforated ribbing is confirmed by the experimental data of [20] on heat exchange in a stationary annular channel, for which the heat-transfer coefficients were increased by 100%.

The organization of the jet motion within the cylinder on the scheme described above allows one to retain rather high heat-transfer coefficients with an increase in the rotation rate, whereas with fully filled channels and partially filled channels without turbulization by jets the heat-transfer coefficients are considerably decreased [12] or pass through minima [2, 4], depending on the mode of flow of the axial stream.

It should be noted that the method of separation of the total coefficient of the rate of the process into partial components using optimization of the parameters of the assumed functions was also proposed for the study of mass transfer [21] (without the development of numerical methods of solution, unfortunately) and can be applied to many other processes having characteristics which are difficult to measure.

NOTATION

B, coefficient to Eq. (3); b, width of outer annular gap; c, sum of thermal resistances of heat transfer by Eq. (2); C_p, heat capacity; G, mass flow rate of heat-transfer agent; l, length of cylinder; K, heat-transfer coefficient; m, number of series of tests; n, number of tests in a series; $Nu = 2\alpha' b/\lambda$, Nusselt number; q, heat flux density; r, radius of cylinder; $Re = 2ub/\nu$, Reynolds number of axial stream; T, t, average temperature of heat-transfer agent; $Ta = \omega (r')^{1/2} b^{3/2}/\nu$, Taylor number; u, average velocity of axial stream in outer annular channel; α, heat-transfer coefficient; β_c, coefficient to Eq. (5); β_λ, coefficient to Eq. (4); δ/λ_δ, thermal resistance of wall; λ, coefficient of thermal conductivity; μ, coefficient of dynamic viscosity; ν, coefficient of kinematic viscosity; ω, angular rotation rate of cylinder. Indices: i, serial number of tests in a series; j, serial number of series of tests; ', annular channel containing inner rotating cylinder; ", channel within rotating cylinder; wa, at wall temperature; *, heat transfer through annular channel from wall of rotating cylinder to wall of stationary cylinder (and vice versa); —, arithmetic mean value for α'' and root-mean-square value for ε.

LITERATURE CITED

1. F. Kreitz, "Convective heat exchange in rotating systems," in: Progress in Heat Transfer, Vol. 5 [Russian translation], Mir, Moscow (1971), p. 144.
2. V. K. Shchukin, Heat Exchange and Hydrodynamics of Internal Flows in Mass Force Fields [in Russian], Mashinstroenie, Moscow (1970).
3. P. N. Singh and D. N. Ghosh, Indian Chem. Eng., 11, No. 1, 29 (1969).
4. R. F. Pattenden, J. Mech. Eng. Sci., 6, No. 3, 309 (1964).
5. F. Hettler and W. Spormann, "Vorrichtung zum Abscheiden fester Stoffe aus Lösungen [Badische Anilin & Soda-Fabrik A.-G.]," FRG Patent No. 1,059,883, March 12 (1959).
6. A. G. Bol'shakov, G. M. Dolgikh, G. G. Mikhailenko, F. V. Kozak, A. V. Skorichenko, N. F. Shapa, and B. S. Kavnat-skaya, "Crystallizer," Author's Certificate No. 263,561, Byull. Izobr., No. 8 (1970).
7. A. G. Bol'shakov, G. M. Dolgikh, F. K. Mikhailov, and S. M. Dolbenskii, "Reactor," Author's Certificate No. 316,465, Byull. Izobr., No. 3 (1971).
8. J. Taborek, T. Aoki, R. B. Ritter, J. W. Palen, and J. G. Knudsen, Chem. Eng. Progr., 68, No. 2, 59 (1972).
9. J. Taborek, T. Aoki, R. B. Ritter, J. W. Palen, and J. G. Knudsen, Chem. Eng. Progr., 68, No. 7, 69 (1972).
10. J. Taborek, T. Aoki, R. B. Ritter, and J. W. Palen, Amer. Inst. Chem. Eng. Symp. Ser., 68, No. 118, 45 (1972).
11. L. A. Dorfman, Hydrodynamic Resistance and Heat Transfer of Rotating Bodies [in Russian], Fizmatgiz, Moscow (1960).
12. V. M. Buznik, Intensification of Heat Exchange in Ship Installations [in Russian], Sudostroenie, Leningrad (1969).
13. V. N. Zmeikov and B. P. Ustimenko, in: Problems of Thermal Power Engineering and Applied Heat Physics [in Russian], Part 1, Alma-Ata (1964), p. 148.
14. S. I. Kosterin and Yu. P. Finat'ev, Inzh.-Fiz. Zh., 5, No. 8, 3 (1962).
15. F. Tachibana and S. Fukui, Bull. Jap. Soc. Mech. Eng., 7, No. 26, 385 (1964).
16. G. P. Zedgenidze, Measurement of the Temperature of Rotating Machinery Components [in Russian], Mashgiz, Moscow (1962).
17. J. E. R. Coney, J. Mech. Eng. Sci., 6, No. 2, 144 (1964).
18. M. A. Mikheev, Principles of Heat Transfer [in Russian], Gosénergoizdat, Moscow–Leningrad (1947).
19. A. I. Leont'ev and A. N. Kirdyashkin, Inzh.-Fiz. Zh., 13, No. 6, 821 (1967).
20. J. Wisniak and C. Reinel, Canad. J. Chem. Eng., 50, No. 1, 136 (1972).
21. V. I. Konovalov, in: Proceedings of Tambov Institute of Chemical Mechanical Engineering [in Russian], Part 1, Tambov (1967), p. 364.